Determining the Shortest Distance Using Fuzzy Triangular Method

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ABSTRACT
Determining the shortest distance is important in real system. There are some methods to find the distance in a path network. In this paper, a well-known fuzzy triangular method is used and it reduces the error for finding shortest distance in a path network as comparable to other methods. A triangular membership function has been used for fuzzy logic. An ideal fuzzy set has been used to find the degree of membership function, which shows the existence of the shortest and longest path. From these two paths, the proposed FTM selects the shortest path. For each triangle, the FTM selects the shortest path. Eventually, the FTM can find the shortest path to go from a source node to a destination node in any path network. The performance of the FTM has been compared with that of the Bellman dynamic programming approach (BDPA) for obtaining the shortest path in a same path network. The comparison shows zero error in finding shortest distance using both the FTM and BDPA. However, the FTM requires less time compared to BDPA to find the shortest path.

Keywords: Ideal fuzzy set, fuzzy triangular method, Bellman dynamic programming approach, shortest path.

Introduction

In network optimization, the shortest path problem is one of the most important problems. In transportation problem shortest path is used. Over the past several years finding shortest path in a network has been posed as a subset of other optimization problem. In a transport network the shortest path problem requires determining the shortest route between a source and a destination. In real life application, it is very difficult for decision maker to find the exact value of arc length. For decision making problem of real-world fuzzy set theory has been used. In fuzzy set theory a membership function is used to quantify the fuzzy objectives and constraints. Zimmermann [1] proposed the min operator models using linear membership function to the multi-objective linear programming problem. Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker Bellman and Zadeh [2]. It is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be made from among them. Optimization is a king of decision making, in which decisions have to be taken to optimize one or more objectives under some prescribed set of circumstances. In many real-life situations optimization problems are solved as single objective optimization problems in a deterministic and crisp environment.

For supply chain planning under supply, demand and process uncertainties David Poitra proposed fuzzy optimization. In 1980s fuzzy logic was used in numerous fields of research like industrial manufacturing, automatic control, automobile production, banks, hospitals and libraries etc. In risk assessment, the fuzzy mathematical method has been used. In fuzzy mathematics, multi-index synthetic evaluation has been used in natural disaster studies. Fuzzy synthetic evaluation provides a synthetic evaluation of an object relative to an objective in a fuzzy decision environment with multi criteria.

P. K. De and Amita Bhincher [3] discussed two different methods for the same fuzzy shortest path problems. For that problem, they considered triangular fuzzy number and trapezoidal fuzzy numbers. For the solution, problem was first converted into an equivalent crisp problem and applied the recursion procedure. Amita Bhincher applied Cheng's fuzzy ranking method. Klein [4] proposed a dynamic programming recursion based fuzzy algorithm by applying extension principle which gives dominated path on a acyclic network. for the fuzzy shortest path. For multi-objective nonlinear programming problem with fuzzy parameters, Sakawa et al. [5]

proposed an interactive fuzzy decision-making model using linear and nonlinear membership functions to solve the multi-objective linear programming problem.
Dubois and Prade [6] introduce fuzzy shortest path problem based on multiple labeling approach. Okada and Soper [7] developed an algorithm. They introduced an order relation between fuzzy numbers applying fuzzy min concept. A real-life example has been considered by Amita Binchar [8] from Rajasthan state roadways transport network. She applied dynamical programming approach to find the shortest path in a fuzzy network. She used uncertain edge weights from a source to a destination in a network.
Shortest path can be found from various methods as we discussed. All those methods are time consuming but it is our aim to find shortest path by using less time. FTM approach counts only shortest paths and ignores all other paths. As all other paths are ignored except shortest path, so by this method it can be decided that this method is relatively less time consuming.

Fuzzy theory
A crisp set or a classical set *A* is defined as a collection of well-defined objects. The objects are called element of *A*. A crisp set *A*, defined on the universal set *X*, can also be represented by
$A = \{(x, \mu_A(x)) : x \in X\}$
where $\mu_A(x)$ is called characteristic function that declares which element of universal set *X* are member of a set and which are not. Set *A* is defined by its characteristic function $\mu_A(x)$ as

$$\mu_A(x) = \begin{cases} 1, ifx \in A \\ 0, ifx \notin A \end{cases}$$

The characteristic function $\mu_A(x)$ maps elements of set *X* to elements of set {0, 1}, which is formally expressed as
$\mu_A(x) : X \to \{0,1\}$
The characteristic function of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in *X*. This function can be generalized to a function $\bar{\mu}_A$ such that the value assigned to the element of the universal set *X* fall within a specified range [0, 1] i.e. $\bar{\mu}_A = X \to [0,1]$ The assigned values indicate the membership grade of the element in the set *A*. The function $\bar{\mu}_A$ is called the membership function and the set $\tilde{A} = \{(x, \bar{\mu}_A) : x \in X\}$ defined by for each $x \in X$ is called a fuzzy set. $\bar{\mu}_A$ is the degree of membership of *x* in $\tilde{A}$ .The closer the value of is to 1, the more *x* belongs to *A*.

3. Shortest path
A shortest path or geodesic path between two nodes in a graph is a path with the minimum number of edges. The length of a geodesic path is called geodesic distance or shortest distance. Shortest path problem is one of network optimization problems that aims to define the shortest path from one node to another. In graph theory, the shortest path problem is the problem of finding a path between two vertices in a graph such that the sum of the weights of it constituent edges is minimized. The field of network optimization concerns optimization problems on networks. Networks are everywhere

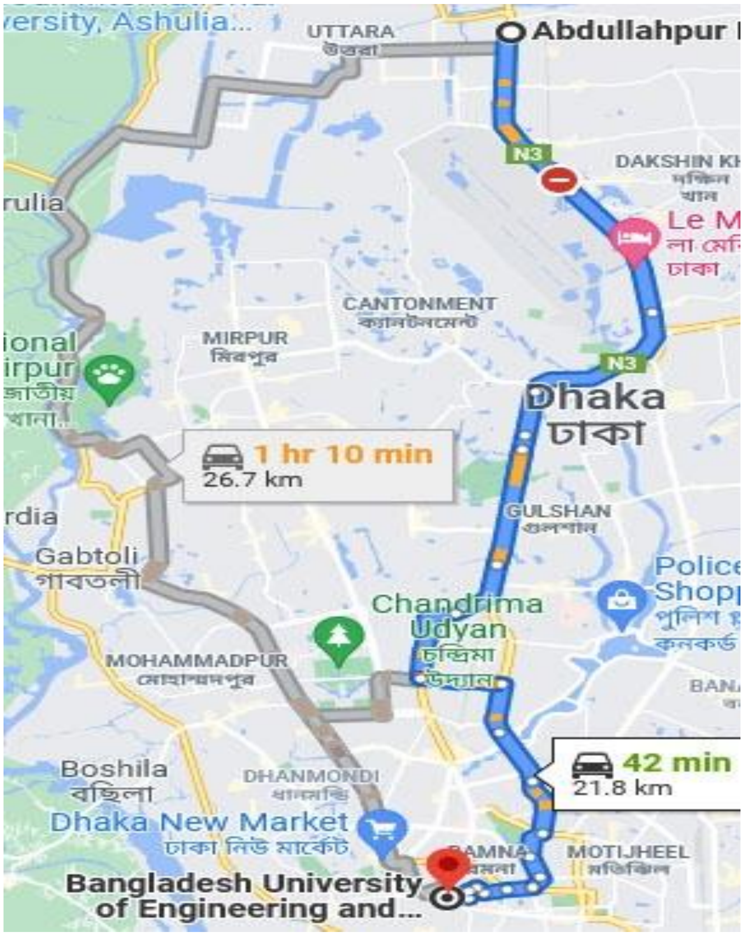
Physical networks	Social networks
1. road networks	1.organizational charts
2. railway networks	2. friendship network
3. airline traffic network	3. interaction network (cell calls)
4. electrical networks	
5.communication networks	

3.1 Ideal fuzzy set

The ideal fuzzy set (IFS) is a set with shorter and longer length having the maximum and minimum degree of membership function respectively. As we want to find the shortest path, the length of the path is very important. Hence the maximum and minimum membership degrees are assigned to shorter and longer lengths of the IFS respectively.

The membership function of IFS is $\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ path\ , x_n =$ largest path

3.2 Finding shortest path



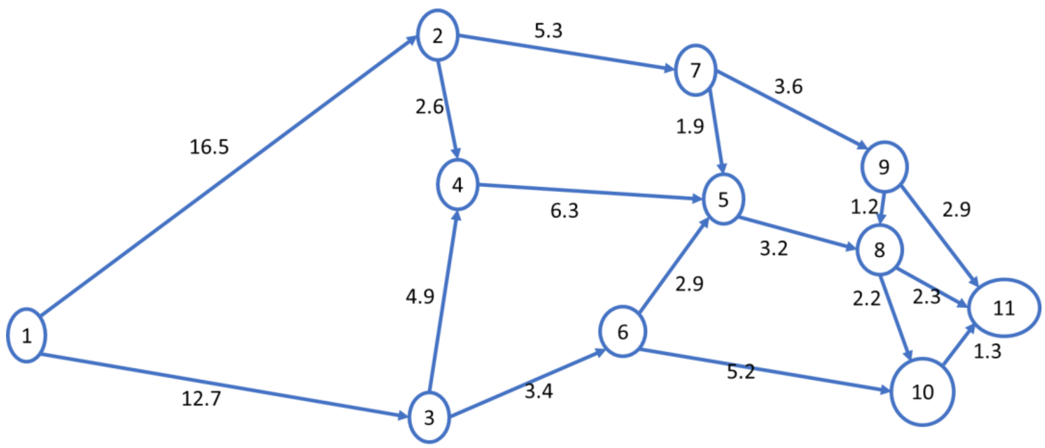
A student of Bangladesh University of Engineering & Technology (BUET) wants to join the class at 8 a.m. by riding bike. The source place is Abdullahpur and the destination place is Polashi, BUET. There are 11 vertices and there are 16 possible paths. The vertices are

1. Abdullahpur 2. Konabari 3. Bonani 4. Mirpur 10 5. Farmgate 6. Mohakhali 7. Asadgate 8. shabag 9. Sciencelab 10. Karjoinhall 11. BUET.

All the possible path ways are given below

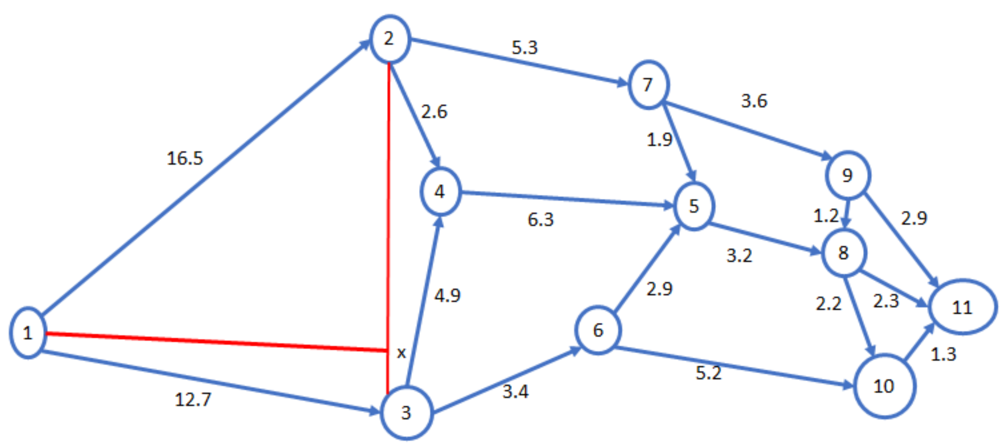
1. $1 \to 2 \to 7 \to 9 \to 11$
2. $1 \to 2 \to 4 \to 5 \to 8 \to 11$
3. $1 \to 2 \to 7 \to 5 \to 8 \to 11$
4. $1 \to 2 \to 7 \to 9 \to 8 \to 11$
5. $1 \to 2 \to 4 \to 5 \to 7 \to 9 \to 11$
6. $1 \to 2 \to 4 \to 5 \to 8 \to 9 \to 11$
7. $1 \to 3 \to 6 \to 10 \to 11$

8. $1 \to 3 \to 4 \to 2 \to 7 \to 9 \to 11$
9. $1 \to 3 \to 4 \to 5 \to 7 \to 9 \to 11$
10. $1 \to 3 \to 4 \to 5 \to 8 \to 9 \to 11$
11. $1 \to 3 \to 4 \to 5 \to 8 \to 11$
12. $1 \to 3 \to 6 \to 5 \to 7 \to 9 \to 11$
13. $1 \to 3 \to 6 \to 5 \to 8 \to 9 \to 11$
14. $1 \to 3 \to 6 \to 5 \to 8 \to 11$
15. $1 \to 3 \to 6 \to 10 \to 8 \to 9 \to 11$
16. $1 \to 3 \to 6 \to 10 \to 8 \to 11$



In triangle 123, $L_{13} = 12.7, L_{12} = 16.5, \mu(12.7) = ?\ \mu(16.5) = ?$ In this triangle degree of shortest path is 1. $\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ \ path\ (12),$ x_n =largest path (17)

we have to find the degree of membership function for existing path network. Between them which length gives the maximum degree of membership that line gives the shortest path. We have to check number of vertices in each and every path. If we get single vertex we have to count next vertex.



$\mu(12.7) = 0.88, \mu(16.5) = 0.25$

$L_1 = max\{\mu(12.7), \mu(16.5)\} = max\{0.88, 0.25\} = 0.88, \quad SL_1 = 12.7km$

In triangle 346, $L_{36} = 3.4, L_{34} = 4.9, \mu(3.4) = ?\ \mu(4.9) = ?$

$\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ \ path\ (3),$ x_n =largest path (5)

$\mu(3.4) = 0.86, \mu(4.9) = 0.36$
$L_2 = max\{\mu(3.4), \mu(4.9)\} = max\{0.86, 0.36\} = 0.86, \quad SL_2 = 3.4km$

In triangle 6105, $L_{65} = 2.9, L_{610} = 7.2, \mu(2.9) = ?\ \mu(7.2) = ?$
$\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ \ path\ (2),$ x_n =largest path (8)

$\mu(2.9) = 0.87, \mu(7.2) = 0.257$
$L_3 = max\{\mu(2.9), \mu(7.2)\} = max\{0.87, 0.257\} = 0.87, \quad SL_3 = 2.9km$

In triangle 578, $L_{58} = 3.2, L_{59} = 5.5, \mu(3.2) = ?\ \mu(5.5) = ?$
$\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ \ path\ (3),$ x_n =largest path (6)

$\mu(3.2) = 0.95, \mu(5.5) = 0.375$
$L_4 = max\{\mu(3.2), \mu(5.5)\} = max\{0.95, 0.375\} = 0.95, \quad SL_4 = 3.2km$

In triangle 8911, $L_{911} = 2.9, L_{89} = 4.1, \mu(2.3) = ?\ \mu(4.1) = ?$
$\mu(x_i) = \frac{x_1 - x_i}{x_n - x_1 + 1} + 1, x_1 = shortest\ \ path\ (2),$ x_n =largest path (5)

$\mu(2.3) = 0.925, \mu(4.1) = 0.475$
$L_5 = max\{\mu(2.3), \mu(4.1)\} = max\{0.925, 0.475\} = 0.925, \quad SL_5 = 2.3 = km$ The shortest path way is $1 \to 3 \to 6 \to 5 \to 8 \to 11$
so the length of the shortest path is=12.7+3.4+2.9+3.2+2.3=24.5 km

3.3 Finding shortest path by using Bellman dynamic programming

$f(1) = 0$
$f(2) = \min\{\ f(1) + c_{12}\} = \min(\ 0 + 16.5) = 16.5$
$f(3) = 12.7,\ f(4) = \min\{\ f(2) + c_{24},\ f(3) + c_{34}\}$
$= \min\{\ 16.5 + 2.6, 12.7 + 4.9\} = \min(\ 19.1, 17.6) = 17.6$

$f(5) = \min\{\ f(4) + c_{45},\ f(6) + c_{65},\ f(7) + c_{75}\} = \min\{\ 17.6 + 6.3, 16.1 + 2.9, 21.8 + 1.9\}$
$= \min\{\ 23.9, 19, 23.7\} = 19$

$f(6) = \ f(3) + c_{36} = 12.7 + 3.4 = 16.1$
$f(7) = \ f(2) + c_{27} = 16.5 + 5.3 = 21.8$
$f(8) = \min\{\ f(5) + 3.2,\ f(10) + 2.2,\ f(9) + 1.2\}$
$= \{19 + 3.2, 23.3 + 2.2, 25.4 + 1.2\}$
$= \{22.2, 25.5, 26.6\} = 22.2$

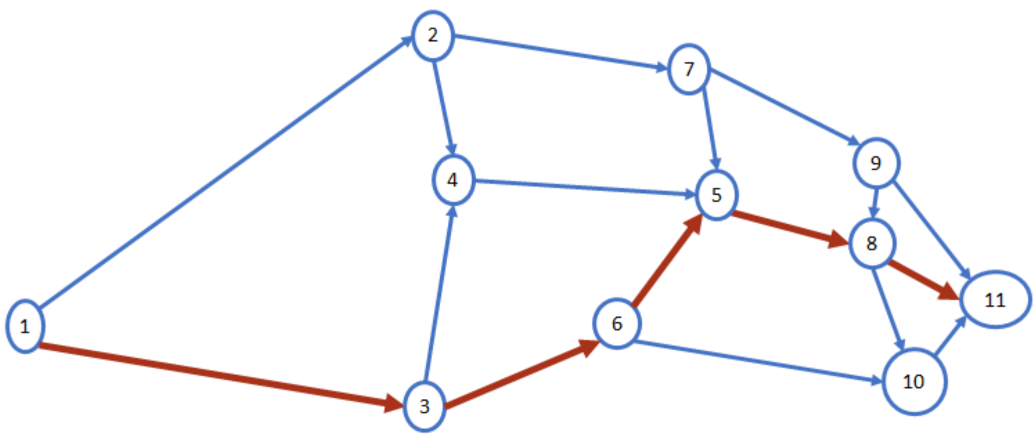
$f(9) = f(7) + 3.6 = 21.8 + 3.6 = 25.4$
$f(10) = f(6) + 7.2 = 16.1 + 7.2 = 23.3$
$f(11) = \min\{ f(8) + 2.3, f(10) + 1.3, f(9) + 2.9\}$
$= \min\{ 22.2 + 2.3, 23.3 + 1.3, 25.4 + 2.9\}$
$= \min\{ 24.5, 24.6, 28.3\} = 24.5$

Therefore, the estimated shortest path is
$1 \to 3 \to 6 \to 5 \to 8 \to 11 = 12.7 + 3.4 + 2.9 + 3.2 + 2.3 = 24.5$ km

3.4 Consistency check

$$\text{Error} = \frac{exact\,value - present\,value}{exact\,value} \times 100\% = \frac{24.5 - 24.5}{24.5} \times 100\% = 0$$

Distance of all possible paths are given below
$1 \to 2 \to 7 \to 9 \to 11 = 16.5 + 5.3 + 3.6 + 2.9 = 28.3$ km
$1 \to 2 \to 4 \to 5 \to 8 \to 11 = 16.5 + 2.6 + 6.3 + 3.2 + 2.3 = 30.9$ km
$1 \to 2 \to 7 \to 5 \to 8 \to 11 = 16.5 + 5.3 + 1.9 + 3.2 + 2.3 = 29.2$ km
$1 \to 2 \to 7 \to 9 \to 8 \to 11 = 16.5 + 5.3 + 3.6 + 1.2 + 2.3 = 28.9$ km
$1 \to 2 \to 4 \to 5 \to 7 \to 9 \to 11 = 16.5 + 2.6 + 6.3 + 1.9 + 3.6 + 2.9 = 33.8$ km
$1 \to 2 \to 4 \to 5 \to 8 \to 9 \to 11 = 16.5 + 2.6 + 6.3 + 3.2 + 1.2 + 2.9 = 32.7$ km
$1 \to 3 \to 6 \to 10 \to 11 = 12.7 + 3.4 + 7.2 + 1.3 = 24.6$ km
$1 \to 3 \to 4 \to 2 \to 7 \to 9 \to 11 = 12.7 + 4.9 + 2.6 + 5.3 + 3.6 + 2.9 = 32$ km
$1 \to 3 \to 4 \to 5 \to 7 \to 9 \to 11 = 12.7 + 4.9 + 6.3 + 1.9 + 3.6 + 2.9 = 32.3$ km
$1 \to 3 \to 4 \to 5 \to 8 \to 9 \to 11 = 12.7 + 4.9 + 6.3 + 3.2 + 1.2 + 2.9 = 31.2$ km
$1 \to 3 \to 6 \to 10 \to 11 = 12.7 + 3.4 + 7.2 + 1.3 = 29.4$ km
$1 \to 3 \to 6 \to 5 \to 7 \to 9 \to 11 = 12.7 + 3.4 + 2.9 + 1.9 + 3.6 + 2.9 = 27.4$ km
$1 \to 3 \to 6 \to 5 \to 8 \to 9 \to 11 = 12.7 + 3.4 + 2.9 + 3.2 + 1.2 + 2.9 = 26.3$ km
$1 \to 3 \to 6 \to 5 \to 8 \to 11 = 12.7 + 3.4 + 2.9 + 3.2 + 2.3 = 24.5$ km
$1 \to 3 \to 6 \to 10 \to 8 \to 9 \to 11 = 12.7 + 3.4 + 7.2 + 2.2 + 1.2 + 2.9 = 29.6$ km
$1 \to 3 \to 6 \to 10 \to 8 \to 11 = 12.7 + 3.4 + 7.2 + 2.2 + 2.3 = 27.8$ km



4 Conclusion

For shortest path network a lot of works has been published. In this paper FTM method has been applied finding shortest path for any path network. An example has been given from Abdullahpur (North Dhaka) to BUET (South Dhaka) for finding shortest path in Dhaka city. For checking Bellman dynamic programming approach has also been applied for the same path network. It can be seen that FTM gives same shortest distance as BDPA.

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